PRIOR KNOWLEDGE

MATHEMATICS

FORM 3 : CHAPTER 1
INDICES

PRIOR KNOWLEDGE
MATHEMATICS FORM 3 : CHAPTER 1

1.1 INDEX NOTATION

a^n — Index (n), Base (a)

REMINDER

$2^5 \neq 2 \times 5$

$4^3 \neq 4 \times 3$

$a^n \neq a \times n$

$$a^n = \underbrace{a \times a \times a \times \ldots \times a}_{n \text{ factors}} ;\ a \neq 0$$

Recap 1:

Write the following repeated multiplications in index for a^n.

$\underbrace{5 \times 5 \times 5 \times 5 \times 5 \times 5}_{\text{Repeated 6 times}} = \mathbf{5^6}$

Recap 2:

Write 64 in index form using of base of 2, base of 4 and base of 8.

$64 = \mathbf{2^6}$

$64 = \mathbf{4^3}$

$64 = \mathbf{8^2}$

4.1 LAWS OF INDICES

INDICES (PRIOR KNOWLEDGE)

1. $a^m \times a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$
3. $(a^m)^n = a^{mn}$

Example : Law 1

Multiplication of numbers in index form	Repeated Multiplication
$2^3 \times 2^4$	$(2\times2\times2)\times(2\times2\times2\times2) = 2\times2\times2\times2\times2\times2\times2 = 2^7$

Example : Law 2

Division of numbers in index form	Repeated Multiplication
$4^5 \div 4^2$	$\frac{4^5}{4^2} = \frac{4\times4\times4\times4\times4}{4\times4} = 4^3$

Example : Law 3

Multiplication of numbers in index form	Repeated Multiplication
$(3^2)^4$	$3^2 \times 3^2 \times 3^2 \times 3^2 = 3^{2+2+2+2} = 3^8$

REMARKS

$a^m \times b^m = (ab)^m$

$\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

INDICES (PRIOR KNOWLEDGE)

4. $a^0 = 1$
5. $a^{-m} = \frac{1}{a^m}$

Example : 4

Division in Index Form	Solution		Conclusion From The Solution
	Law of Indices	Repeated Multiplication	
$2^3 \div 2^3$	$2^{3-3} = 2^0$	$\frac{2\times2\times2}{2\times2\times2} = 1$	$2^0 = 1$

Example : 5

Division in Index Form	Solution		Conclusion From The Solution
	Law of Indices	Repeated Multiplication	
$2^3 \div 2^5$	$2^{3-5} = 2^{-2}$	$\frac{2\times2\times2}{2\times2\times2\times2\times2} = \frac{1}{2^2}$	$2^{-2} = \frac{1}{2^2}$

TIPS

$a^n = \frac{1}{a^{-n}}$

$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

SIMPLIFYING ALGEBRAIC EXPRESSIONS INVOLVING INDICES

Example 1: Simplify the following algebraic expressions.

(a) $\dfrac{5^{3x} \times 5^{x}}{5^{-x}}$

Solution:

$$\frac{5^{3x} \times 5^{x}}{5^{-x}} = 5^{3x+x-(-x)}$$

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

$$= 5^{5x}$$

(b) $\dfrac{9^{a-3} \times 9^{a+4}}{81}$

Solution:

$$\frac{9^{a-3} \times 9^{a+4}}{81} = \frac{9^{a-3} \times 9^{a+4}}{9^{2}}$$

Use base 9 for all terms

$$= 9^{a-3+a+4-2}$$

$$= 9^{2a-1}$$

Add and substract the indices using the laws of indices

SIMPLIFYING ALGEBRAIC EXPRESSIONS INVOLVING INDICES

Example 2: Simplify the following algebraic expressions.

$$\sqrt[5]{a^7} \times \sqrt[4]{a^{-9}}$$

Solution:

$$\sqrt[5]{a^7} \times \sqrt[4]{a^{-9}} = a^{\frac{7}{5}} \times a^{-\frac{9}{4}}$$

$$\sqrt[m]{a^n} = a^{\frac{n}{m}}$$

$$= a^{\frac{7}{5}+\left(-\frac{9}{4}\right)}$$

$$a^m \times a^n = a^{m+n}$$

$$= a^{-\frac{17}{20}}$$

SIMPLIFYING ALGEBRAIC EXPRESSIONS INVOLVING INDICES

Example 3:

Show that

(a) $4^{3a-2} = \frac{64^a}{16}$

(b) $9^{2a+2} = 81(81^a)$

Solution:

(a) $\underbrace{4^{3a-2}}_{\text{LHS}} = \underbrace{\frac{64^a}{16}}_{\text{RHS}}$

$\text{LHS} = 4^{3a-2}$

$= \frac{4^{3a}}{4^2}$

$a^m \div a^n = a^{m-n}$

$= \frac{64^a}{16}$

$= \text{RHS}$

Solution:

(b) $\underbrace{9^{2a+2}}_{\text{LHS}} = \underbrace{81(81^a)}_{\text{RHS}}$

$9^{2a+2} = 9^{2a} \times 9^2$

$a^m \times a^n = a^{m+n}$

$= (81^a)(81)$

2 x 4 = 8
4 x 2 = 8

$= 81(81^a)$

$= \text{RHS}$

SIMPLIFYING ALGEBRAIC EXPRESSIONS INVOLVING INDICES

Example 4:

Show that $5^{n+2} - 7(5^n) - 25(5^{n-1})$ is divisible by 13 for all positive integers of n.

Solution:

$$5^{n+2} - 7(5^n) - 25(5^{n-1}) = 5^n \times 5^2 - 7(5^n) - 5^2\left(\frac{5^n}{5^1}\right)$$

$$= 5^n\left(5^2 - 7 - 5\right)$$

$$= 5^n(13)$$

divisible by 13 for all positive integers of n

Use base 5 for all terms

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

Factorise completely

SIMPLIFYING ALGEBRAIC EXPRESSIONS INVOLVING INDICES

Example 5:

If $\dfrac{5^{6n}\times 9^{4n}\times 15^{2n}}{3^{2n}}=k^{8n}$, where k is a positive integer, find the value of k.

Solution:

$$\frac{5^{6n}\times 9^{4n}\times 15^{2n}}{3^{2n}}=\frac{5^{6n}\times(3^2)^{4n}\times(5\times 3)^{2n}}{3^{2n}}$$

Use base 3 and 5

$$=\frac{5^{6n}\times 3^{8n}\times 5^{2n}\times 3^{2n}}{3^{2n}}$$

$a^m\times a^n=a^{m+n}$

$a^m\div a^n=a^{m-n}$

$$=5^{6n+2n}\times 3^{8n+2n-2n}$$

$$=5^{8n}\times 3^{8n}$$

$$=(5\times 3)^{8n}$$

$a^m\times b^m=(a\times b)^m$

$$=15^{8n}$$

Hence, by comparing, $k = 15$

SIMPLIFYING ALGEBRAIC EXPRESSIONS INVOLVING INDICES

Example 6:

Given that $p = 3^x$ and $q = 3^y$. Express $\dfrac{27^{x+y}}{9^x}$ in terms of p and q.

Solution:

$$\frac{27^{x+y}}{9^x} = \frac{(3^3)^{x+y}}{3^{2x}}$$

Use base 3 for all terms

$$= \frac{(3^{3x})\times(3^{3y})}{3^{2x}}$$

$a^m \times a^n = a^{m+n}$

$$= \frac{(3^{x3})\times(3^{y3})}{3^{x2}}$$

$$= \frac{(3^x)^3(3^y)^3}{(3^x)^2}$$

$$= \frac{p^3q^3}{p^2}$$

$$= pq^3$$

SOLVING PROBLEMS INVOLVING INDICES

Equations involving indices can be solved as follows:

1. If $a^x = a^y$, then $x = y$
2. If $a^x = b^x$, then $a = b$

when a > 0 and a ≠ 1

Example 7: Solve each of the following equations.

(a) $2^{3x} = 12 - 2^{3x-1}$

Solution:

$$2^{3x} + 2^{3x-1} = 12$$

$$2^{3x} + \frac{2^{3x}}{2^1} = 12$$

$a^m \div a^n = a^{m-n}$

$$2^{3x}\left(1 + \frac{1}{2}\right) = 12$$

Factorise completely

$$2^{3x}\left(\frac{3}{2}\right) = 12$$

$$2^{3x} = 8$$

$$2^{3x} = 2^3$$

Use base 2 for all terms

$$3x = 3$$

By comparing index

$$x = 1$$

(b) $8(4^x) = \frac{32}{16^{1-2x}}$

Solution:

$$8(4^x) = \frac{32}{16^{1-2x}}$$

$$(4^x) \times (16^{1-2x}) = \frac{32}{8}$$

$$(4^x) \times (4^{2(1-2x)}) = 4^1$$

Use base 4 for all terms

$$4^{x+2-4x} = 4^1$$

$a^m \times a^n = a^{m+n}$

$$2 - 3x = 1$$

By comparing index

$$x = \frac{1}{3}$$

SOLVING PROBLEMS INVOLVING INDICES

Example 8:

In a research, a type of bacteria will multiply itself in 1 minute. The amount of bacteria at the start of the research was 300. The amount of bacteria after t minutes is given by $300(3^t)$.

(a) Determine the amount of bacteria after 9 minutes.

(b) Determine the time, t, in minutes for the amount of the bacteria to be 72 900.

Solution:

(a) Amount of Bacteria, $B = 300(3^t)$

when $t = 9$, $B = 300(3^9)$

$= 5\,904\,900$

(b) $300(3^t) = 72\,900$

$3^t = 243$

$3^t = 3^5$ By comparing index

$t = 5$

4.2 LAWS OF SURDS

RATIONAL NUMBERS AND IRRATIONAL NUMBERS

Rational Numbers

- Numbers can be written in $\frac{p}{q}$ fraction, where p and q are integers and $q \neq 0$.

INTEGERS

$6=\frac{6}{1}, 13=\frac{13}{1}$

TERMINATING DECIMALS

0.8, 0.75, 4.086

RECURRING DECIMALS

0.2828..., 11.868686...

Irrational Numbers

- Numbers in the form of **non-recurring decimals** and **infinite decimals**.

$\pi = 3.141592654...$

$e = 2.718281828...$

SURD

$\sqrt{13}, \sqrt{5}$

Irrational number in the form of root

RATIONAL NUMBERS AND IRRATIONAL NUMBERS

Number	Simplified Number	Decimal	Surd or Not Surd
$\sqrt{3}$	$\sqrt{3}$	1.7320508…	Surd
$\sqrt{\frac{1}{4}}$	$\frac{1}{2}$	0.5	Not Surd
$\sqrt[3]{11}$	$\sqrt[3]{11}$	2.2239800…	Surd
$\sqrt[3]{27}$	3	3	Not Surd
$\sqrt[5]{3}$	$\sqrt[5]{3}$	1.2457309…	Surd

Extra Input

$\sqrt[n]{x}$ is read as 'surd x order n'

$\sqrt[3]{5}$ is read as 'surd 5 order 3'

RATIONAL NUMBERS AND IRRATIONAL NUMBERS

Example 9: Convert the following recurring decimals to a fraction.

(a) 0.292929292929…

Solution:

(a) Let

$$N = 0.292929292929\ldots \quad \textbf{(1)}$$

$$100N = 29.2929292929\ldots \quad \textbf{(2)}$$

There is two digits repeating. Multiply both sides of equation by 100

(2) – (1) : $99N = 29$

$$N = \frac{29}{99}$$

Thus, $0.292929292929\ldots = \dfrac{29}{99}$

(b) 13.567567567…

Solution:

(b) Let

$$A = 13.567567567\ldots$$

$$A = 13 + 0.567567\ldots$$

$$A = 13 + N$$

Assume,

$$N = 0.567567567\ldots \quad \textbf{(1)}$$

$$1000N = 567.567567\ldots \quad \textbf{(2)}$$

There is three digits repeating. Multiply both sides of equation by 1000

(2) – (1) : $999N = 567$

$$N = \frac{567}{999}$$

$$A = 13 + \frac{567}{999}$$

Thus, $13.567567567\ldots = 13\dfrac{567}{999}$

SIMPLIFYING EXPRESSIONS INVOLVING SURDS

Laws of Surds

Law 1 : $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$

Law 2 : $\sqrt{a}\div\sqrt{b}=\sqrt{\frac{a}{b}}$

Example 10: Simplify each of the following

(a) $\sqrt{50}$

Solution:

$$\sqrt{50}=\sqrt{25\times2}$$

$$=\sqrt{25}\times\sqrt{2}$$

$\sqrt{a}\times\sqrt{b}=\sqrt{ab}$

$$=5\sqrt{2}$$

(b) $\sqrt{98p}\div\sqrt{2p}$

Solution:

$$\sqrt{98p}\div\sqrt{2p}=\frac{\sqrt{98p}}{\sqrt{2p}}$$

$$=\sqrt{\frac{98p}{2p}}$$

$\sqrt{a}\div\sqrt{b}=\sqrt{\frac{a}{b}}$

$$=\sqrt{49}$$

$$=7$$

SIMPLIFYING EXPRESSIONS INVOLVING SURDS

$$m\sqrt{a} \pm n\sqrt{a} = (m \pm n)\sqrt{a}$$

Example 11: Simplify the following expressions.

(a) $9\sqrt{7} - 2\sqrt{7}$

Solution:

$$9\sqrt{7} - 2\sqrt{7} = (9-2)\sqrt{7}$$
$$= 7\sqrt{7}$$

(b) $\sqrt{32} + 3\sqrt{2}$

Solution:

$$\sqrt{32} + 3\sqrt{2} = \sqrt{16 \times 2} + 3\sqrt{2}$$
$$= \sqrt{16} \times \sqrt{2} + 3\sqrt{2}$$

$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$$= 4\sqrt{2} + 3\sqrt{2}$$
$$= (4+3)\sqrt{2}$$

$m\sqrt{a} \pm n\sqrt{a} = (m \pm n)\sqrt{a}$

$$= 7\sqrt{2}$$

SIMPLIFYING EXPRESSIONS INVOLVING SURDS

Example 12: Simplify the following expressions.

(a) $\sqrt{6}(3\sqrt{6}-5\sqrt{6})$

Solution:

$$\sqrt{6}(3\sqrt{6}-5\sqrt{6}) = \sqrt{6}\,(-2\sqrt{6}\,)$$
$$= -12$$

$\sqrt{a}\times\sqrt{a}=a$

(b) $(9+5\sqrt{4})(3-5\sqrt{4})$

Solution:

$$(9+5\sqrt{4})(3-5\sqrt{4})$$
$$=27-45\sqrt{4}+15\sqrt{4}-25(4)$$
$$=27+(-45+15)\sqrt{4}-100$$
$$=27-100-30\sqrt{4}$$
$$=-73-30\sqrt{4}$$
$$=-133$$

$m\sqrt{a}\pm n\sqrt{a}=(m\pm n)\sqrt{a}$

SMART TIPS

$$\sqrt{a}\times\sqrt{a}=a^{\frac{1}{2}}\times a^{\frac{1}{2}}$$
$$=a^{\frac{1}{2}+\frac{1}{2}}$$
$$=a^{1}$$

RATIONALISING THE DENOMINATORS FOR EXPRESSIONS INVOLVING SURDS

- Multiply the numerator and denominator of $\frac{1}{m\sqrt{a}}$ with the conjugate surd $m\sqrt{a}$ so that the surd can be eliminated.
- Multiply the numerator and denominator of $\frac{1}{m\sqrt{a}+n\sqrt{b}}$ with the conjugate surd $m\sqrt{a}-n\sqrt{b}$ so that the surd can be eliminated.
- Multiply the numerator and denominator of $\frac{1}{m\sqrt{a}-n\sqrt{b}}$ with the conjugate surd $m\sqrt{a}+n\sqrt{b}$ so that the surd can be eliminated.

SMART TIPS

$$\sqrt{a}\times\sqrt{a}=a^{\frac{1}{2}}\times a^{\frac{1}{2}}=a$$

$$(a-b)(a+b)=a^2-b^2$$

Surd	Conjugate Surd
$m\sqrt{a}$	$m\sqrt{a}$
$m\sqrt{a}+n\sqrt{b}$	$m\sqrt{a}-n\sqrt{b}$
$m\sqrt{a}-n\sqrt{b}$	$m\sqrt{a}+n\sqrt{b}$

RATIONALISING THE DENOMINATORS FOR EXPRESSIONS INVOLVING SURDS

Example 13: Simplify the following.

(a) $\dfrac{5}{\sqrt{3}}$

Solution:

(a) $\dfrac{5}{\sqrt{3}} = \dfrac{5}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$

Multiply with the conjugate surd

$= \dfrac{5\sqrt{3}}{3}$

(b) $\dfrac{7}{2-\sqrt{5}}$

Solution:

(b) $\dfrac{7}{2-\sqrt{5}} = \dfrac{7}{(2-\sqrt{5})} \times \dfrac{(2+\sqrt{5})}{(2+\sqrt{5})}$

Multiply with the conjugate surd

$= \dfrac{14+7\sqrt{5}}{(2^2-5)}$

$= \dfrac{14+7\sqrt{5}}{-1}$

$= -14-7\sqrt{5}$

(c) $\dfrac{2\sqrt{3}}{\sqrt{3}+\sqrt{2}}$

Solution:

(c) $\dfrac{2\sqrt{3}}{\sqrt{3}+\sqrt{2}} = \dfrac{2\sqrt{3}}{(\sqrt{3}+\sqrt{2})} \times \dfrac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2})}$

$= \dfrac{2(3)-2\sqrt{3}\sqrt{2}}{(3-2)}$

$= 6-2\sqrt{6}$

$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

SMART TIPS

$\sqrt{a} \times \sqrt{a} = a^{\frac{1}{2}} \times a^{\frac{1}{2}} = a$

$(a-\sqrt{b})(a+\sqrt{b}) = a^2 - b$

RATIONALISING THE DENOMINATORS FOR EXPRESSIONS INVOLVING SURDS

Example 14:

Rationalise the denominator and simplify $\dfrac{3+\sqrt{2}}{5-\sqrt{5}}$.

Solution:

$$\frac{3+\sqrt{2}}{5-\sqrt{5}} = \frac{(3+\sqrt{2})}{(5-\sqrt{5})} \times \frac{(5+\sqrt{5})}{(5+\sqrt{5})}$$

Multiply with the conjugate surd

$$= \frac{15+3\sqrt{5}+5\sqrt{2}+\sqrt{2}\sqrt{5}}{25-5}$$

$$= \frac{15+3\sqrt{5}+5\sqrt{2}+\sqrt{10}}{20}$$

$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

SMART TIPS

$\sqrt{a} \times \sqrt{a} = a^{\frac{1}{2}} \times a^{\frac{1}{2}} = a$

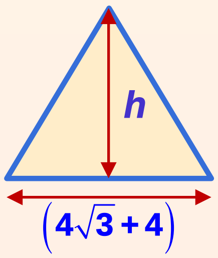
$(a-b)(a+b) = a^2 - b^2$

SOLVING PROBLEMS INVOLVING SURDS

Example 15:

The diagram on the right shows a pyramid- shaped house. The triangle shape at the front of the house has an area of $\left(20\sqrt{3}-4\right)$ m² and the length of its base is $\left(4\sqrt{3}+4\right)$m. Determine the height of the triangle at the front of the house in the form of $\left(a+b\sqrt{3}\right)$, where a and b are rational numbers.

Solution:



$$\text{Area}=\left(20\sqrt{3}-4\right)$$

$$\frac{1}{2}\times\left(4\sqrt{3}+4\right)\times h=20\sqrt{3}-4$$

$$\left(4\sqrt{3}+4\right)\times h=40\sqrt{3}-8$$

$$\frac{4\sqrt{3}+4}{4\sqrt{3}+4}\times h=\frac{40\sqrt{3}-8}{4\sqrt{3}+4}$$

$$h=\frac{40\sqrt{3}-8}{4\sqrt{3}+4}\times\frac{4\sqrt{3}-4}{4\sqrt{3}-4}$$

Multiply with the conjugate surd

$$=\frac{160(3)-160\sqrt{3}-32\sqrt{3}+32}{16(3)-16}$$

$$=\frac{512-192\sqrt{3}}{32}$$

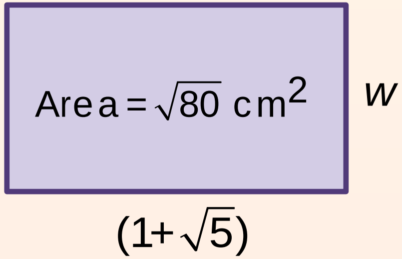
$$h=16-6\sqrt{3}$$

$$\text{Area}=\frac{1}{2}\times\text{base}\times\text{height}$$

Example 16:

A carpet with a rectangle shape has a length of $(1+\sqrt{5})$ cm and an area of $\sqrt{80}$ cm^2. calculate the width of the carpet, w in cm. Express your answer in the form of $a+b\sqrt{5}$, where a and b are integers to be found.

Solution:



$$\text{Area} = \sqrt{80}$$

$$(1+\sqrt{5}) \times w = \sqrt{80}$$

$$\frac{(1+\sqrt{5})w}{(1+\sqrt{5})} = \frac{\sqrt{80}}{(1+\sqrt{5})}$$

$$w = \frac{\sqrt{80}}{(1+\sqrt{5})} \times \frac{(1-\sqrt{5})}{(1-\sqrt{5})}$$

$$= \frac{\sqrt{80} - \sqrt{80}\sqrt{5}}{1-5}$$

$$w = \frac{\sqrt{16\times 5} - \sqrt{400}}{-4}$$

$$= \frac{4\sqrt{5} - 20}{-4}$$

$$= -\sqrt{5} + 5$$

Hence, $a = 5$, $b = -1$

4.3 LAWS OF LOGARITHMS

Relating The Equations in Index Form With Logarithmic Form and Determining The Logarithmic Value of A Number

For a positive number $a \neq 1$

$$a^x = N \leftrightarrow \log_a N = x$$

Example 17:

Convert the following to logarithmic form.

(a) $6^2 = 36$

Solution:

(a) $6^2 = 36$

$\log_6 36 = 2$

(b) $10x = y^3$

Solution:

(b) $10x = y^3$

$\log_y 10x = 3$

Example 18:

Convert the following to index form.

(a) $\log_8 64 = 2$

Solution:

(a) $\log_8 64 = 2$

$8^2 = 64$

(b) $\log_{10} \frac{1}{1000} = -3$

Solution:

(b) $\log_{10} \frac{1}{1000} = -3$

$10^{-3} = \frac{1}{1000}$

Relating The Equations in Index Form With Logarithmic Form and Determining The Logarithmic Value of A Number

Example 19:

Convert $2 = 5^m$ and $3 = 7^n$ to logarithmic form. Hence, express each of the following in terms of m and/or n.

(a) $\log_5 2 + \log_7 7 - \log_5 1$

(b) $\log_7 3 - \log_5 2 - 10$

Solution:

$2 = 5^m \Leftrightarrow \log_5 2 = m$

$3 = 7^n \Leftrightarrow \log_7 3 = n$

$a^x = N \Leftrightarrow \log_a N = x$

(a) $\log_5 2 + \log_7 7 - \log_5 1$

$\log_n n = 1$ $\log_n 1 = 0$

$= m + 1 - 0$

$= m + 1$

(b) $\log_7 3 - \log_5 2 - 10$

$= n - m - 10$

NOTES

- Since $a^1 = a$, $\log_a a = 1$
- Since $a^0 = 1$, $\log_a 1 = 0$

PROVING THE LAWS OF LOGARITHMS

Let $x = a^p$ and $y = a^q$, then $p = \log_a x$ and $q = \log_a y$

PRODUCT LAW

(a) $\log_a xy = \log_a x + \log_a y$

Solution:

(a) $xy = a^p \times a^q$

$xy = a^{p+q}$ $a^m \times a^n = a^{m+n}$

Thus, $\log_a xy = p + q$

$\log_a xy = \log_a x + \log_a y$

DIVISION LAW

(b) $\log_a \frac{x}{y} = \log_a x - \log_a y$

Solution:

(b) $\frac{x}{y} = \frac{a^p}{a^q}$

$\frac{x}{y} = a^{p-q}$ $a^m \div a^n = a^{m-n}$

Thus, $\log_a \frac{x}{y} = p - q$

$\log_a \frac{x}{y} = \log_a x - \log_a y$

POWER LAW

(c) $\log_a x^n = n \log_a x$

Solution:

(c) $x^n = (a^p)^n$

$x^n = a^{pn}$ $(a^m)^n = a^{mn}$

Thus, $\log_a x^n = pn$

$\log_a x^n = n \log_a x$

THE LAWS OF LOGARITHMS

Example 20: Given $\log_a 2 = 0.558$ and $\log_a 7 = 1.206$, find the value of each of the following.

(a) $\log_a 28$

Solution:

$$\text{(a) } \log_a 28 = \log_a (7 \times 4)$$
$$= \log_a 7 + \log_a 4$$

$\log_a xy = \log_a x + \log_a y$

$$= \log_a 7 + \log_a 2^2$$
$$= \log_a 7 + 2\log_a 2$$

$\log_a x^n = n\log_a x$

$$= 1.206 + 2(0.558)$$
$$= 2.322$$

(b) $\log_a \frac{2}{7}$

Solution:

$$\text{(b) } \log_a \frac{2}{7} = \log_a 2 - \log_a 7$$

$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

$$= 0.558 - 1.206$$
$$= -0.648$$

(c) $\log_a \sqrt[3]{7}$

Solution:

$$\text{(c) } \log_a \sqrt[3]{7} = \log_a 7^{\frac{1}{3}}$$

$a^{\frac{1}{m}} = \sqrt[m]{a}$

$$= \frac{1}{3}\log_a 7$$
$$= \frac{1}{3}(1.206)$$
$$= 0.402$$

THE LAWS OF LOGARITHMS

Example 21: Express the following as single logarithms.

(a) $\log_m 5+\log_m 8-\log_m 4$

Solution:

(a) $\log_m 5+\log_m 8-\log_m 4$

$=\log_m\left(\frac{5\times 8}{4}\right)$

$\log_a xy=\log_a x+\log_a y$

$\log_a\left(\frac{x}{y}\right)=\log_a x-\log_a y$

$=\log_m \frac{40}{4}$

$=\log_m 10$

(b) $\frac{1}{2}\log_4 x+2-3\log_4 y$

Solution:

(b) $\frac{1}{2}\log_4 x+2-3\log_4 y$

$=\log_4 x^{\frac{1}{2}}+2\times\log_4 4-\log_4 y^3$

$\log_n n=1$

$=\log_4 x^{\frac{1}{2}}+\log_4 4^2-\log_4 y^3$

$=\log_4\left(\frac{x^{\frac{1}{2}}\times 16}{y^3}\right)$

$\log_a xy=\log_a x+\log_a y$

$\log_a\left(\frac{x}{y}\right)=\log_a x-\log_a y$

THE LAWS OF LOGARITHMS

Example 22: Given $\log_a 5 = p$ and $\log_a 3 = q$, express each of the following in terms of p and q.

(a) $\log_a 15$

Solution:

(a) $\log_a 15 = \log_a (3 \times 5)$

$\log_a xy = \log_a x + \log_a y$

$= \log_a 3 + \log_a 5$

$= q + p$

(b) $\log_a 0.6a$

Solution:

(b) $\log_a 0.6a = \log_a \left(\frac{3}{5} \times a \right)$

$= \log_a \frac{3}{5} + \log_a a$

$\log_n n = 1$

$= \log_a 3 - \log_a 5 + 1$

$\log_a \left(\frac{x}{y} \right) = \log_a x - \log_a y$

$= q - p + 1$

CHANGE OF BASES OF LOGARITHMS

Example 23:

1. $\log_a b = \dfrac{\log_c b}{\log_c a}$
2. $\log_a b = \dfrac{1}{\log_b a}$

Given $\log_3 2 = t$, express each of the following in terms of t

(a) $\log_2 9$

(b) $\log_2 \dfrac{9}{4}$

Solution:

(a)
$$\log_2 9 = \frac{\log_3 9}{\log_3 2}$$
$$= \frac{\log_3 3^2}{\log_3 2}$$
Change to base 3
$$= \frac{2\log_3 3}{\log_3 2}$$
$\log_n n = 1$
$$= \frac{2}{t}$$

(b)
$$\log_2 \frac{9}{4} = \log_2 9 - \log_2 4$$
$\log_a\left(\dfrac{x}{y}\right) = \log_a x - \log_a y$
$$= \frac{\log_3 9}{\log_3 2} - \frac{\log_3 4}{\log_3 2}$$
Change to base 3
$$= \frac{\log_3 3^2}{t} - \frac{\log_3 2^2}{t}$$
$$= \frac{2}{t} - \frac{2t}{t}$$
$$= \frac{2}{t} - 2$$

SOLVING PROBLEM INVOLVING THE LAWS OF LOGARITHMS

Example 24:

Solve the equation $4^{2x-1} = 7^x$ by giving answers in three decimal places.

Solution:

$$4^{2x-1} = 7^x$$

$$\log_{10} 4^{2x-1} = \log_{10} 7^x$$

$$(2x-1)\log_{10} 4 = x\log_{10} 7$$

$\log_a x^n = n\log_a x$

$$(2x-1)\frac{\log_{10} 4}{\log_{10} 4} = x\frac{\log_{10} 7}{\log_{10} 4}$$

$$2x-1 = 1.4037x$$

$$2x - 1.4037x = 1$$

$$x = 1.6770$$

SOLVING PROBLEM INVOLVING THE LAWS OF LOGARITHMS

Example 25:

Solve the equation $\log_3 5x - \log_x(2x-1) = 2$

Solution:

METHOD 1

$$\log_3 5x - \log_x(2x-1) = 2$$

$$\log_3\left(\frac{5x}{2x-1}\right) = 2$$

$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

$$\frac{5x}{2x-1} = 3^2$$

$a^x = N \Leftrightarrow \log_a N = x$

$$\frac{5x}{2x-1} \times (2x-1) = 9(2x-1)$$

$$5x = 18x - 9$$

$$x = \frac{9}{13}$$

METHOD 2

$$\log_3\left(\frac{5x}{2x-1}\right) = 2$$

$$\log_3\left(\frac{5x}{2x-1}\right) = 2 \times \log_3 3$$

$$\log_3\left(\frac{5x}{2x-1}\right) = \log_3 3^2$$

$\log_a x^n = n\log_a x$

$$\frac{5x}{2x-1} = 3^2$$

$$5x = 18x - 9$$

$$x = \frac{9}{13}$$

SOLVING PROBLEM INVOLVING THE LAWS OF LOGARITHMS

Example 26:

Solve the equation $\log_3 x - \log_x 81 = 0$

Solution:

$$\log_3 x - \log_x 81 = 0$$

$$\log_3 x - \log_x 81 + \log_x 81 = 0 + \log_x 81$$

$$\log_3 x = \log_x 81$$

$$\log_3 x = \frac{\log_3 81}{\log_3 x}$$

$$\log_a b = \frac{\log_c b}{\log_c a}$$

$$\log_3 x(\log_3 x) = \frac{\log_3 81}{\log_3 x} \times (\log_3 x)$$

$$(\log_3 x)^2 = \log_3 3^4$$

$$(\log_3 x)^2 = 4$$

$$\log_3 x = \sqrt{4}$$

$$\log_3 x = -2,\ 2$$

$$\log_3 x = -2 \qquad \log_3 x = 2$$

$$x = 3^{-2} \qquad x = 3^2$$

$$x = \frac{1}{9} \qquad x = 9$$

APPLICATIONS OF INDICES, SURDS AND LOGARITHMS

Example 27:

A company's saving after n years is RM2 000$(1+0.07)^n$. Determine the minimum number of years for their savings to exceed RM4 000.

Solution:

Company's saving = RM2 000$(1+0.07)^n$

Exceed RM4 000

After n years, company's saving > 4000

$$2\,000(1+0.07)^n > 4\,000$$

$$\frac{2\,000(1+0.07)^n}{2\,000} > \frac{4\,000}{2\,000}$$

$$(1+0.07)^n > 2$$

$$\log_{10}(1+0.07)^n > \log_{10}2$$

$$n\log_{10}(1+0.07) > \log_{10}2$$

$$\frac{n\log_{10}(1+0.07)}{\log_{10}(1+0.07)} > \frac{\log_{10}2}{\log_{10}(1+0.07)}$$

$$n > 10.2448$$

$$n = 11$$

SUMMARY OF CHAPTER 4

INDICES, SURDS AND LOGARITHMS

INDICES

Laws of Indices

1. $a^m \times a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$
3. $(a^m)^n = a^{mn}$

SURDS

1. $\sqrt{a} \times \sqrt{a} = a$
2. $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
3. $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$
4. $(a + \sqrt{b}) \times (a - \sqrt{b})$
   $= a^2 - b$

LOGARITHMS

1. $\log_a xy = \log_a x + \log_a y$
2. $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$
3. $\log_a x^n = n \log_a x$
4. $\log_a b = \frac{\log_c b}{\log_c a}$
5. $\log_a b = \frac{1}{\log_b a}$

APPLICATIONS

INDICES

- ✓ Geometric Progressions
- ✓ Simultaneous Equations
- ✓ Linear Law

SURDS

- ✓ Trigonometry
- ✓ Coordinate Geometry
- ✓ Vector
- ✓ Calculus

LOGARITHMS

- ✓ Geometric Progressions
- ✓ Linear Law